Using architecture as a context to enhance students' understanding of symmetry


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Working within the context of a contest to find the most beautiful tourist attraction in the world, Year 5 students investigate the most symmetric building from a short-list of three iconic buildings. This involves investigating both line and rotational symmetry in order to make an informed decision about the symmetrical nature of each building.

What is symmetry?

Symmetry is a fundamental part of nature and is visible in many aspects of life including architecture, art and design. In geometry, symmetry means that the characteristics of a shape are unaffected by transformations such as reflection, rotation or translation.

Line symmetry

In daily conversation, the word symmetry generally refers to reflective symmetry (also called mirror or line symmetry). In this article, we use the term line symmetry due to its common use in the curriculum (Australian Curriculum and Assessment Reporting Authority (ACARA), 2014). A shape has line symmetry if a line can be drawn through it which divides the shape into two halves which are mirror images of each other (Van de Walle, 2010). For example, an isosceles triangle (see Figure 1) has one line of symmetry. However, a shape may have a number of lines of symmetry. For example, a square has four lines of symmetry (see Figure 2). Lines of symmetry may be described as vertical, horizontal or diagonal.

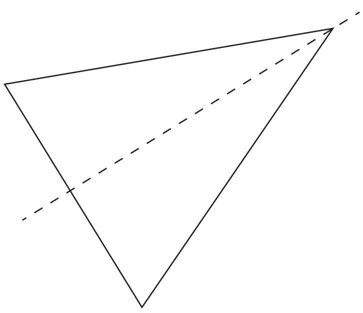


Figure 1. Line symmetry in an isosceles triangle.

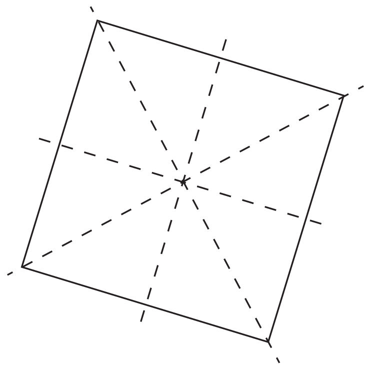


Figure 2. Lines of symmetry in a square.

Rotational symmetry

Another form of symmetry is rotational symmetry (also called point symmetry). A shape has rotational symmetry if, when it is rotated about a fixed point, it lands in a position which matches the original shape. If a shape matches its original shape twice in one full (360°) rotation then its rotational symmetry is described as 'order 2'. Therefore, a regular hexagon (see Figure 3) has rotational symmetry of order 6 because, in one full rotation, the shape matches the original six times (Van de Walle, 2010).

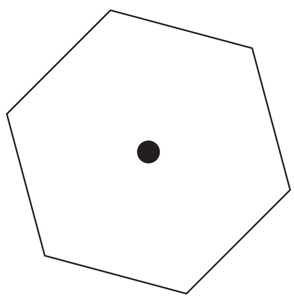


Figure 3. Regular hexagon with rotational symmetry of order 6.

Symmetry in the primary mathematics classroom

Within the Australian primary curriculum strand of measurement and geometry, and specifically the sub-strand of location and transformation, symmetry is introduced from Year 3 where it is recommended that students "identify symmetry in the environment" (ACMMG066). In Year 4 students' understandings are developed further through opportunities to "create symmetrical patterns, pictures and shapes with and without digital technologies" (ACMMG091). In Year 5, students are expected to "describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries"(ACMMG 114; ACARA, 2014).

Hence, symmetry is an integral component of the geometry curriculum as it supports students in focusing on the characteristics and parts of an object. Furthermore, symmetry connects mathematics to the real world as symmetry can be found in everyday items ranging from the natural world of animals, plants and flowers to human made constructions in the fields of art and architecture. Thus it comes as no surprise that teachers frequently draw on these contexts when teaching symmetry. Indeed, research indicates that relevant situational contexts can be used to motivate, to illustrate potential applications, to promote mathematical reasoning and thinking, and to situate student understanding (Sparrow, 2008). Teachers have successfully used contexts which highlight symmetry in everyday situations through trademarks (Renshaw, 1986), children's literature (Harris, 1998), flags (Dolkino, 1996) and flowers (Gavin, 2001; Seidel, 1998).

This paper presents an example of one context we found particularly useful in our teaching of symmetry, the use of architecture.

Judging the most symmetric international attraction

What follows is an overview of the outcomes of teaching a sequence of instruction focusing on symmetry for Year 5 students (10–12 year olds). By a sequence of instruction, we mean 2–3 mathematics lessons focusing on the concept. Central to the teaching was the critical role played by the context of architecture and how it was used to stimulate interest, maintain engagement and support students in developing understandings of symmetry alongside the proficiencies of reasoning and problem solving (ACARA, 2014; Leavy and Hourigan, 2015; Sparrow, 2008). In our study, the sequence was as follows:

- **Lesson 1:** introduction to the context and developing understanding of line symmetry.
- **Lesson 2:** developing understandings of rotational symmetry.
- **Lesson 3:** consolidating understanding of symmetry concepts by applying knowledge to the context.

The scenario: Students were introduced to Tripadvisor, an online forum used to submit and read reviews about accommodation, restaurants and attractions. They were presented with the scenario that Tripadvisor was running a competition to choose the most beautiful international tourist attraction and they were part of a final judging panel. In order to enter the competition, tourist attractions had to meet a specific criterion—they must have symmetry. Students were informed that many people believe that symmetric objects appear more aesthetically beautiful than asymmetric objects (i.e., objects with no line or rotational symmetry).

The task: Their task, as judges, was to identify the most symmetric building. They were told that the Tripadvisor website had shortlisted three attractions:

1. Blue Mosque, Istanbul
2. Taj Mahal, India
3. Chichen Itza, Mexico

As the competition was focusing on each of the attraction's line and rotational symmetry from an aerial or 'bird's-eye view' (see Figure 4), the teacher used Google Earth to locate each attraction geographically and provide a brief virtual tour around its exterior (see Figure 5). Therefore, in order to act as judges and provide an objective and fair judgement, students needed to learn about both types of symmetry.

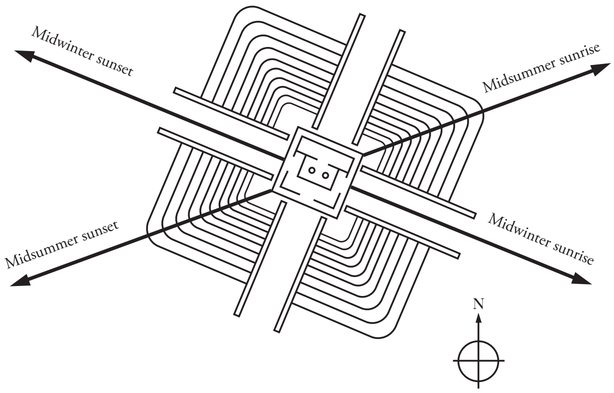


Figure 4. A bird's-eye view of Chichen Itza, Mexico.

Figure 5. An exterior view of Chichen Itza.

Developing students' concept of symmetry

When asked to share their understanding of the term symmetry, it was clear that students' prior knowledge was solely related to line symmetry:

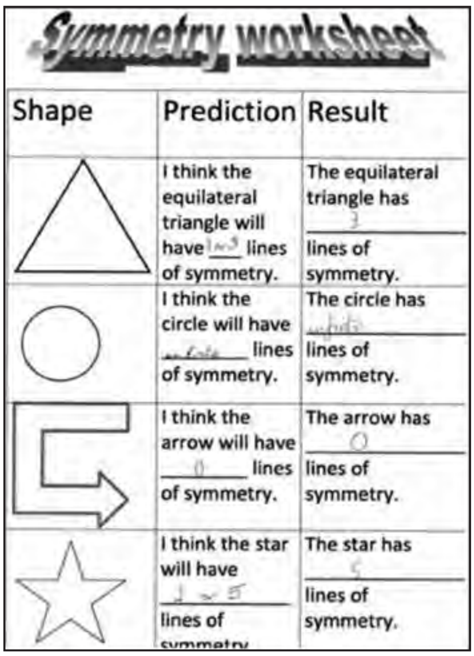
Betsy: Symmetry is when we fold over a shape and it will be the same size when you fold it over…so there are no parts sticking out. Both sides are the same.

Line symmetry

Initial activities focused on revising line symmetry. The teacher challenged the class to identify the number of lines of symmetry within a square: "Who wants to come up and show me a line of symmetry?" The teacher focused on the various types of line symmetry, for example "...Paschal folded the square across and it folded exactly onto itself. So a square has a line of symmetry. Take a look at it. How would we describe this line of symmetry?" (Answer: vertical). The class were then challenged to discover the other lines of symmetry (i.e., horizontal and diagonal line symmetry).

Despite much exposure to this concept in the past, one student when demonstrating did not unfold the shape before folding again. The teacher used this as a teaching point: "When you are showing lines of symmetry you must start again from the original shape. Otherwise you are looking at the line symmetry of a different shape".

The class was divided into pairs to predict and test line symmetry in a selection of shapes. The activity sheet focused students on four shapes (see Figure 6) and was used to record predictions and findings. In each case, students received a paper cut-out of the shape (see Figure 7) which encouraged them to make and test predictions (through folding).



Symmetry worksheet

Shape	Prediction	Result
	I think the equilateral triangle will have [illegible] lines of symmetry.	The equilateral triangle has 3 lines of symmetry.
	I think the circle will have [illegible] lines of symmetry.	The circle has [illegible] lines of symmetry.
	I think the arrow will have 0 lines of symmetry.	The arrow has 0 lines of symmetry.
	I think the star will have [illegible] lines of symmetry.	The star has 5 lines of symmetry.

Figure 6. Line symmetry activity sheet.

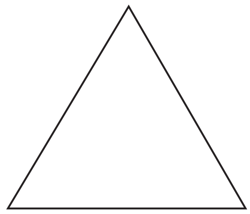


Figure 7. Paper cut-out shape.

However, in order to promote development of flexible understandings of line symmetry, it is recommended that non-prototypical depictions (orientation) of regular shapes (see Figures 1 and 2) are presented, as it encourages students to search for all possible lines of symmetry. It is also advisable to use a variety of polygons (both irregular and regular) as well as other 2-D shapes (see Figures 6 and 8) (Hourigan and Leavy, 2015).

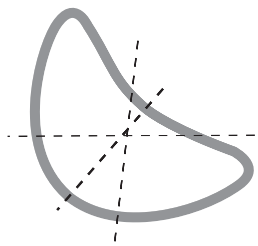


Figure 8. Testing lines of symmetry in kidney bean shape.

Simple strategies, such as tracing over the fold lines, focused students on alternative lines of symmetry and accurate counting. While all students successfully identified the number of lines of symmetry in each shape, these Year 5 students were more accurate when using shape manipulation (as compared to visualising the shape on the activity sheet). Observations revealed some interesting trends:

- **Line symmetry of the circle:** While many students were aware that the circle had "…never ending amount of lines of symmetry", "so many we can't count", the majority seemed to be unfamiliar with the term 'infinite'.
- **Underestimation of line symmetry:** Students' ability to predict the number of lines of symmetry was limited (see triangle and star in figure 6). It appeared that they could only recognise the vertical line of symmetry of the shape and needed prompting to rotate the cut-out shapes and consider others.

The authors, on reflection, considered the potential use of a 'Mira' (a geometric tool which has both reflective and transparent qualities which facilitates the exploration of symmetry) as an alternative to paper folding. The advantage of the 'Mira' is that it assists the student to determine all lines of symmetry without rotating the shape. It also moves the students' focus away from the vertical line of symmetry.

Rotational symmetry

Rotational symmetry was a new concept for students. Their attention was drawn to the classroom clock on the wall:

> "Take a look at the clock. What is the second hand of the clock doing? "

Children were quick to respond that "it rotates". The teacher then ensured that all students understood the meaning of 'rotate': "Rotate means move around a point. So the second hand of the clock moves around the centre of the clock".

Rotational symmetry was initially summarised as follows: "When a shape is rotated around its centre point, the number of times the shape fits exactly on itself, in one full rotation, is called the shape's order of rotational symmetry".

This definition was accompanied by demonstration using a poster with the shape outline displayed on it and a cut-out of the identical shape. A mark was made on both the poster and cut-out shape (see Figure 9, arrow visible on shape) in order to identify the starting and finishing point of a full rotation. The teacher focused on the need to align the two marks (i.e., starting point). In order to keep the class actively involved, they were asked to say "stop" every time the shape fitted on itself. Using a pencil to hold the centre point (see Figure 10), the teacher made one full rotation (360°; until the marks match again), counting aloud the number of times that the shape fitted exactly onto itself. The clarity of the teacher demonstration and explanation were found to be directly related to students' levels of success in subsequent activities. For example, following the exploration of rotational symmetry of a rectangle the teacher stated: "We rotated the rectangle and it fitted onto itself twice. This means it has rotational symmetry of order two".

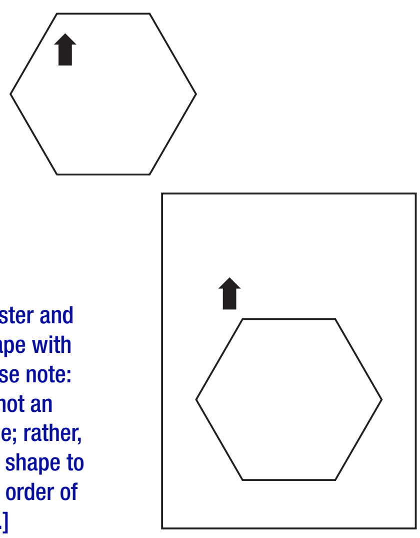


Figure 9. Sample poster and identical cut-out shape with starting points [Please note: the mark (arrow) is not an attribute of the shape; rather, it was placed on the shape to aid determination of order of rotational symmetry.]

Figure 10. Finding the order of rotational symmetry.

The class was then encouraged to predict the order of rotational symmetry of the isosceles triangle. The teacher then tested predictions. This shape was selected for demonstration as it only fitted on itself once in a complete 360° rotation i.e., rotational symmetry of order one. Students were alerted to the convention that if a shape fits on itself only once during a full 360° rotation, the shape is considered to have no rotational symmetry.

Students then worked in pairs to predict and test the rotational symmetry of various shapes. They were encouraged to alternate roles, thus facilitating both to have the opportunity to predict, check and record. Each pair initially received an activity sheet (similar to that used for line symmetry, Figure 6). At this stage, they only completed the prediction section. Subsequently, large cut-out shapes and their matching background shapes were distributed to facilitate testing of initial predictions (see Figure 11). It is important to include irregular polygons as well as other 2-D shapes (see Figures 6, 8 and 12). Students correctly identified the rotational symmetry of the various shapes. Errors were minor, with pairs predicting that the order of rotational symmetry for the irregular shape was 0, rather than 1.

Figure 11. Finding the rotational symmetry of the hexagon.

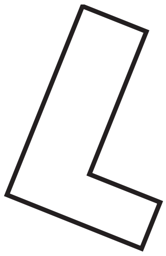


Figure 12. Irregular hexagon.

Consolidating understanding

Students were given the outline shape for each of the three tourist attractions shortlisted for the Tripadvisor competition. These outline shapes were paper cut-outs of the bird's-eye view of the attraction. Working in pairs, they examined each attraction's outline shapes for line symmetry, followed by class discussion and demonstration to confirm the number of lines of symmetry (see Figure 13).

Figure 13. Folding to examine the line symmetry of the bird's-eye view of Chichen Itza, Mexico.

Students' attention was then drawn to the rotational symmetry within each attraction's outline shape. Once again, students used knowledge gained about this concept to predict and check the order of rotational symmetry for the three attractions (see Figure 14).

Figure 14. Rotating the cut-out of the Chichen Itza bird's-eye view to identify rotational symmetry.

All information students had gathered regarding the attractions' symmetry was then considered and used to select and justify the attraction they felt demonstrated most symmetry and thus should win the competition. Each student was asked to write a brief report, in the guise of a Tripadvisor reviewer, about which attraction they would select and why, referring to symmetry.

Extending the activity

There is much potential to adjust the activities to provide more support or challenge as appropriate. To support some learners, the teacher may need to use whole class demonstration to examine the symmetry evident in buildings prior to commencing the pair work. To provide greater challenge for other learners, consider presenting the following informal conjectures and encourage students to accept or reject the conjecture and provide justifications and examples to support their decisions:

- The line (or rotational) symmetry of a building is always equal to the number of sides. True or false?
- The order of rotational symmetry is always the same as the number of lines of symmetry. True or false?

[Note: We chose these conjectures as students sometimes made these generalisations.]

A further conjecture which would provide challenge is:

- Not all buildings with rotational symmetry will have line symmetry but all buildings with line symmetry will have rotational symmetry. True or false?

Reflections

There is widespread support for the use of relevant situational contexts within the teaching of mathematics (ACARA, 2014; Leavy and Hourigan, 2015; Sparrow, 2008; Van de Walle, 2010). This sequence of instruction explored how a context can be used with Year 5 students to both review the concept of line symmetry as well as introduce rotational symmetry in a meaningful way. Students' learning was motivated by the need to respond to a plausible, yet fictitious, problem. On reflection, it may be valuable to launch the lesson sequence with an exploration of buildings closer to the students' lived experience using Google Earth; for example their school or their local cinema. Exploration of the symmetry within these buildings would provide an appropriate springboard on which to develop the context within this paper. The context also supported a focus on developing proficiencies such as problem solving and reasoning (ACARA, 2014). Higher order thinking was particularly evident in tasks requiring students to make generalisations (e.g., a building's rotational symmetry is often the same as its line symmetry). Overall, this research illustrates that the provision of an interesting context supports the development of understandings of symmetry. It also facilitated the students to further appreciate that symmetry is present in many aspects of the world around them.

References

Australian Curriculum and Assessment Reporting Authority (2014). *Foundation to year 10 curriculum: Mathematics, measurement and geometry.* Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Dolinko, L. (1996). Investigating flags: a multicultural approach. *Teaching Children Mathematics, 3*(4), 186–190.

Fernandez, C. & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning.* Mahwah, NJ: Lawrence Erlbaum Associates.

Gavin M.K. (2001). *Navigating through geometry in grades 3–5.* Reston: NCTM

Hourigan, M. & Leavy, A. (2015). What's a real 2D shape? Designing appropriate geometric instruction. *Australian Primary Mathematics Classroom, 20*(1), 24–29.

Harris, J. (1998). Using literature to investigate transformations. *Teaching Children Mathematics, 4*(9), 510–513

Leavy, A. & Hourigan, M. (2015). Budding architects: Exploring the designs of pyramids and prisms. *Australian Primary Mathematics Classroom, 20*(3), 17–23.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, 22*(4), 12–17, 50–52.

Renshaw, B.S. (1986). Symmetry the trademark way. *Arithmetic Teacher 34* (September), 6–12.

Seidel, J. (1998). Symmetry in season. T*eaching Children Mathematics, 4*(5), 244–246.

Sparrow, L. (2008). Real and relevant mathematicas: Is it realistic in the classroom? *Australian Primary Mathematics Classroom, 13*(2), 4–8

Stigler, J.W., & Hiebert, J. (1999). *The teaching gap.* Chapter 7. New York: Free Press.

Van de Walle, J. A. (2010). *Elementary and middle school mathematics: Teaching developmentally.* 7th ed. New York: Pearson Education.